CH301 Unit 1

GAS LAWS REVIEW: MODELING GASES, GAS LAW STOICHIOMETRY

Goals for Today

- Introduction to gases
  - Define and describe gases qualitatively

- Modeling Gases
  - Discuss the eponymous gas laws
  - Combine the gas laws into the ideal gas law
  - Extend the ideal gas law to quantify density

- Gas Law Stoichiometry
  - Add one layer of complexity to stoichiometry problems

- Maybe: Introduction to Kinetic Molecular Theory
  - Describe the assumptions of KMT
  - Introduce the relationships between kinetic energy, temperature
  - Introduce the relationships between velocity and mass of a gas

The Exam is a Week from Today

- Check the office hours schedule for help, especially if you are struggling with fundamentals

- My office hours/reviews up until the exam:
    - Wednesday, WCH 1.120 1-2pm
    - **Sunday Exam Review, Bur 106 5-6pm** ✓
    - Monday, Bur 108, 3-4pm (last minute questions)

- A student is an "A" student but makes an 86 on exam one. Why?

- A student was top of their high school class but makes a 70 on exam one. Why?

Four conceptual points you NEED to know:

1. Ideal gas law calculations MUST use Kelvin and NOT Celsius

$$^\circ C + 273.15 = K$$

2. Know which R-value to use based on your measurements
   - For the ideal gas law: your R-value is set by your pressure units
   - For Kinetic Molecular Theory: your R-value is in energy units (8.314 J/mol K)

$$atm \rightarrow \frac{L \cdot atm}{mol\ K}$$

$$torr \rightarrow \frac{L\ torr}{mol\ K}$$

3. Know your diatomics: when you read "nitrogen gas," know that this means N_2

$$H_2,\ N_2,\ O_2,\ F_2,\ Cl_2,\ Br_2,\ I_2$$
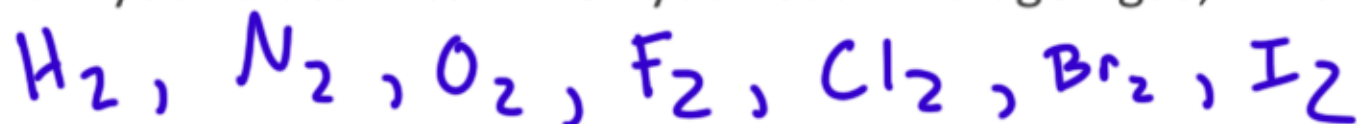
4. The energy of a gas is solely dependent on the temperature

Introduction to Gases

- As an introduction to gases, we pretend that all gases are "ideal gases."

- This comes with a few assumptions:

  - A gas consists of a bunch of molecules that obey Newtonian physics

  - Gas molecules are tiny spheres and they all have the same volume – however, this volume is so small we consider it negligible. **Later on, we will explain that this means there are no repulsive forces present.**

  - No forces act upon these gas molecules except for instantaneous, perfectly elastic collisions – **in particular, gas molecules do not attract each other**
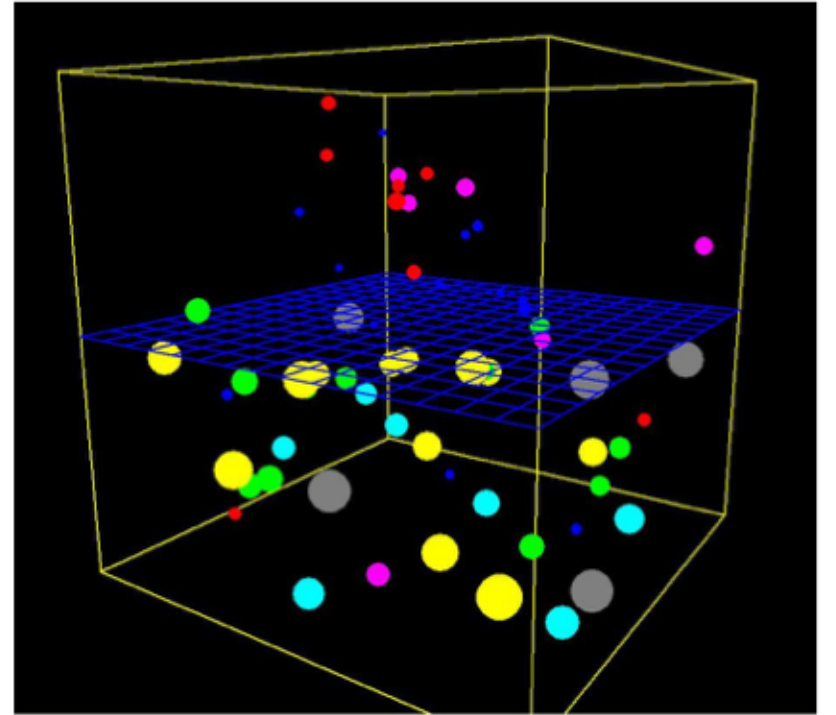
Yes, we will refute these assumptions later.

Modeling Gases

- We describe ideal gases using **state functions**
    - State functions: a quantity or description that explains the current state of a chemical system
    - Another way to describe them is that they explain the final and initial, but not the process in between

- The state functions we use to describe gases are:
    1. Pressure (P): number of **collisions** between the gas and the walls of the container
    2. Volume (V): the volume of the **container**
    3. Temperature (T): **Kelvin**
    4. Number of moles (n): the **quantity** of gas present

$$\hookrightarrow P = \frac{F}{A}$$

Modeling Gases: Common Laws

Boyle's Law

- The pressure and volume of a gas are inversely proportional
- As volume decreases, pressure increases

$$V \uparrow \quad P \downarrow$$

$$P_1 V_1 = P_2 V_2$$
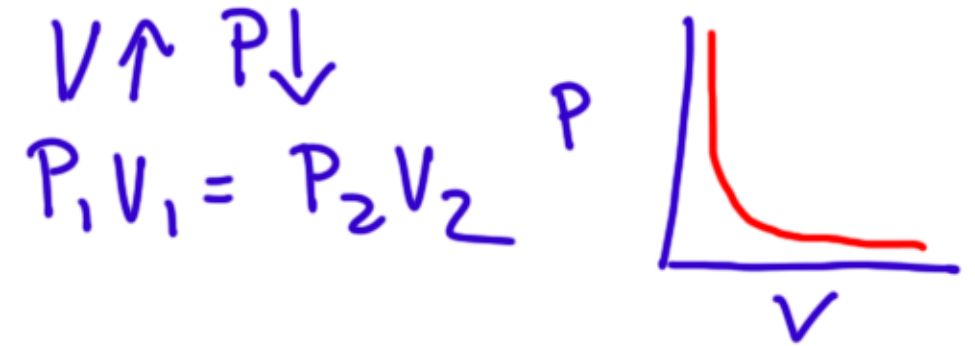
Charles Law

- The volume and temperature of a gas are directly proportional
- As temperature increases, volume increases
- **TEMPERATURE MUST BE IN KELVIN**

$$T \uparrow \quad V \uparrow$$

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

liquid

Avogadro's Law

- The volume and number of moles of a gas are directly proportional
- As you add more gas moles, the volume expands

$$\uparrow n \quad V \uparrow$$

$$\frac{V_1}{n_1} = \frac{V_2}{n_2}$$

Exam-Style Questions

A gas sample is stored in a closed, rigid container at 50°C. How will the pressure change if you increase the temperature to 100°C?

$50 + 273.15 = 323.15$

$100 + 273.15 = 373.15$

1. ~~The pressure will double~~

2. The pressure will increase by a small amount

3. There will be no change in pressure

4. The pressure will halve

5. The pressure will decrease by a small amount

A gas sample has an initial pressure equal to P_i at equilibrium. Which of the following represents the final pressure (P_f) of the gas sample when the volume is quartered (reduced to 1/4 the original volume) at constant temperature?

1. None of the above

2. $P_f = 2P_i$

3. $P_f = 4P_i$

4. $P_f = 0.25P_i$

5. $P_f = 0.5P_i$

6. $P_f = P_i$

$P_i = 1 \text{ atm}$

$V_i \rightarrow \frac{1}{4}V_i$

$4 \text{ atm} = 1 \text{ atm} (4)$

$4L \rightarrow 1L$

$P_f = 4 \text{ atm}$

All Eponymous Gas Laws

Pretty much everything on the exam can be solved using the ideal gas law, but don't forget the individual relationships that make it possible.

- **Boyle's Law**: Pressure and volume have an inverse relationship

$$P \propto \frac{1}{V}$$

- **Charles' Law**: Volume and temperature have a direct relationship

$$V \propto T$$

- **Avogadro's Law**: Volume and number of moles have a direct relationship

$$V \propto n$$

- **Bike Pump Law**: Pressure and number of moles have a direct relationship

$$P \propto n$$

- **Amonton's Law**: Pressure and temperature have a direct relationship

$$P \propto T$$

- **Hard-to-do-this Law**: number of moles and temperature have an inverse relationship

$$n \propto \frac{1}{T}$$

- Or just remember that given PV=nRT:
  - State functions on the same side have an **inverse** relationship
  - State functions on opposite sides have a **direct** relationship

Modeling Gases: The Ideal Gas Law

- The relationship between these state functions is presented in the following equation:

$$PV = nRT$$

direct

inverse

- How we use it: the ideal gas law describes a system at a particular **state** through the four **state** functions: pressure, volume, quantity, and temperature.

- We can also use it for two states:

$$\frac{P_1 V_1}{n_1 T_1} = \frac{P_2 V_2}{n_2 T_2}$$

The Universal Gas Constant - R

PV = nRT

- **The reason why this relationship works as an equality is because of the universal gas constant, R**

- The role of R is to cancel your units. **If R does not match the units of pressure, volume, moles, and temperature, you will end up with the wrong value.**

  - The units of R stem from the following relationship:

$$\frac{PV}{nT} = R$$

  - For this exam, the R-value you choose will be based on your unit of **pressure** in the question

$$\frac{(atm)(L)}{(mol)(K)} = 0.08206\ L \cdot atm \cdot mol^{-1} \cdot K^{-1}$$

$$\frac{(torr)(L)}{(mol)(K)} = 62.364\ L \cdot torr \cdot mol^{-1} \cdot K^{-1}$$

Quick Question

STP is short for "Standard temperature and pressure." For gases this is defined as 0 degrees Celsius and 1 atm pressure. Knowing this, what is the volume of 1 mole gas at STP?

$R = 0.08206$ L atm/mol K

$R = 8.314$ J/mol K

$R = 62.364$ L torr/mol K

Use PV = nRT

= 22.4 L/mol

Note: The value you get for this answer is definitely worth memorizing for the exam.

$$\frac{n}{V} = \text{\# density} \qquad \frac{m}{V} = \text{mass density}$$

$$\xrightarrow{\times MM}$$

The Ideal Gas Law and Density

$$P\cancel{V} = \frac{nRT}{\cancel{V}}$$

$$MM \times P = \frac{n}{V}RT \times MM$$

$$MM \times P = \frac{m}{V}RT$$

$$MM = \frac{\frac{m}{V}RT}{P}$$

$$\frac{MM \times P}{RT} = \frac{m}{V}$$

The Ideal Gas Law and Density

$7\dfrac{m}{V}$

$$\rho = \frac{MM \cdot P}{R \cdot T} \qquad \text{or} \qquad MM = \frac{\rho \cdot R \cdot T}{P}$$

- You can solve for the mass density of a gas using the molar mass (or vice versa) with a simple modification of the ideal gas law.

- What is density?
  - Number density $= \dfrac{n}{V}$
  - Mass density (or just density) $= \dfrac{m}{V} = \rho$

- Two take-away conceptual points:
  - **All other conditions being equal in the ideal gas law equation (P, V, & T), all ideal gases have the same number density;** number density depends on P, V, & T but NOT molecular weight
  - **All other conditions being equal in the ideal gas law equation (P, V, & T), a heavier gas will have a greater mass density;** mass density depends on the molecular weight of the gas

McCordian Question

$$MM = \frac{PRT}{P}$$

You have 2.39 g ideal gas that occupies 3 L at 740 torr and 300K. Which ideal gas is it?

R = 0.08206 L atm/mol K

R = 8.314 J/mol K

R = 62.364 L torr/mol K

$$20.1 \, g/mol = \frac{\left(\frac{2.39 \, g}{3 L}\right)\left(62.36 \frac{L \, torr}{mol \, K}\right)(300 K)}{(740 \, torr)}$$

a. Helium **(4.00 g/mol)**,

b. Neon **(20.1 g/mol)**,

c. Argon **(39.95 g/mol)**,

d. Krypton **(83.8 g/mol)**,

e. Xenon **(131.3 g/mol)**

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Remember from last week…

We have covered the important fundamentals of stoichiometry. However, when studying you should always think: **how can we make these questions more fun** (a.k.a. harder)? Some examples include:

- **Convert your number of moles into mass or weight**
  - Use dimensional analysis to convert from moles to grams to pounds, etc.
  - Dr. B's class: know how to do this by tomorrow!

- **Account for an experimental percent yield**
  - Simply multiply your final answer by the percent yield
  - For example: in the last problem our answer was 2.50 moles of iron (III) oxide. This represents a 100% **theoretical yield**. If the actual percent yield is 50%, your **experimental yield** is 1.25 moles (2.50 x 0.50).

$$\frac{\text{Experimental Yield}}{\text{Theoretical Yield}} \cdot 100\% = \text{Percent Yield}$$
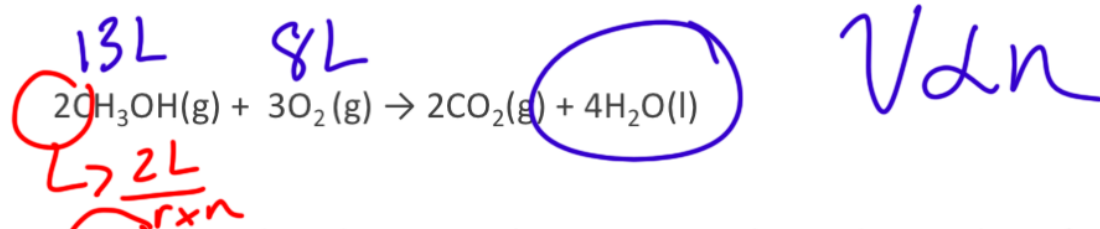
- **Add a gas law problem at the end (we will cover this later in Unit 1)**
  - For example, use the total volume of gases to solve for the total pressure

Stoichiometry Problem 2.0

Consider the combustion of methanol at STP.

$$13L \qquad 8L$$

$2CH_3OH(g) + 3O_2(g) \rightarrow 2CO_2(g) + 4H_2O(l)$

$$\downarrow \frac{2L}{rxn}$$

$V \propto n$

If 13 L of methanol reacts with 8 L oxygen, what is the number of moles of the reactant in excess in the final reaction mixture?

On your own: What is the total volume of all gases? What is the partial pressure of each gas?

$$CH_3OH: \quad 13L \times \frac{rxn}{2L} = 6.5 \, rxn$$

$$O_2: \quad 8L \times \frac{rxn}{3L} = \boxed{2.\overline{66} \, rxn}$$

$$+\frac{mol}{22.4L}$$

$$V_{excess} = V_{initial} - (V_{consumed})$$

$$13L - \left(2.\overline{66} \, rxn \times \frac{2L}{rxn}\right) = \boxed{7.\overline{66} \, L}$$

$$PV = nRT$$

$$n = \frac{PV}{RT} = \frac{(1 \, atm)(7.\overline{66}L)}{(0.08206 \frac{L \cdot atm}{mol \cdot k})(273.15)}$$

$$STP \rightarrow 0°C \rightarrow 273.15$$
$$\rightarrow 1 \, atm$$
$$\rightarrow 1 \, mole \; occupies \; 22.4L$$
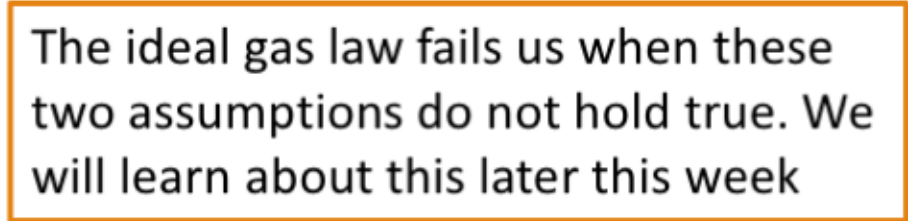
$$= 0.342 \, moles$$

Kinetic Molecular Theory
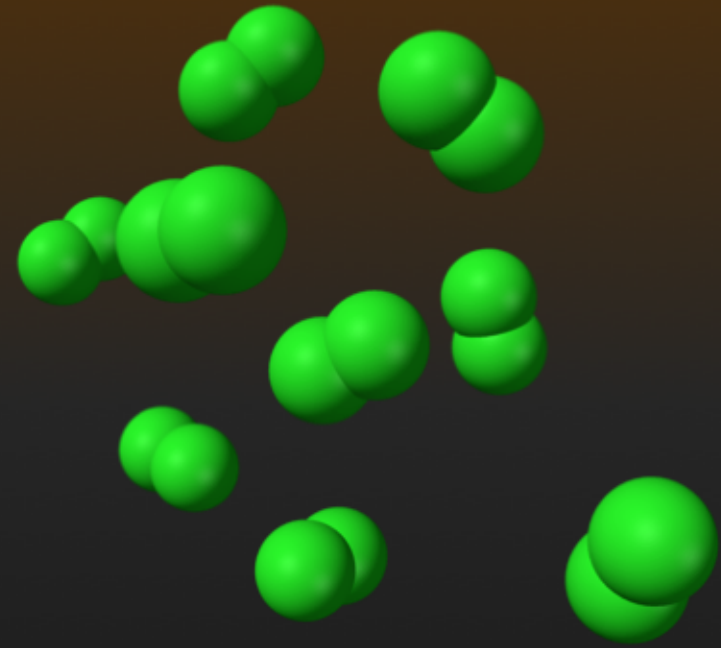
Kinetic Molecular Theory

Kinetic Molecular Theory explains the "energetic" component of gases, including kinetic energy, velocity, and the relationship between kinetic energy and temperature

1. Gases are constantly moving in random directions

2. The distance between particles is large compared to the particle size
   - True ideal gases have relatively **no volume**

3. All particles have perfectly elastic collisions
   - There is no energy loss in the system to collisions; energy cannot be created or destroyed based on Newtonian Physics

4. No other forces act upon ideal gases
   - There are no attractive or repulsive forces that act upon ideal gas particles

The ideal gas law fails us when these two assumptions do not hold true. We will learn about this later this week

Main conclusions: the ideal gas law works when these pillars of KMT hold true in a system. The ideal gas law is modeled best at **High Temperature** and **Low Pressure**

KMT: Conceptual Points

1. Kinetic energy depends ONLY on temperature

$$E_k = \frac{3}{2}RT$$

2. When you have different gases at a given temperature:
   - The heavy gases move slower on average
   - **The lighter gases move faster on average**
   - **All gases have the same kinetic energy**

3. When you have the same gas at different temperatures:
   - The samples have different kinetic energies because the temperatures is different and **kinetic energy depends on temperature**
   - **The hotter gas sample moves faster on average**
   - The colder gas sample moves slower on average

4. Gas velocity is given as a stastistical average of velocities, v_{rms}
   - Gases move at a distribution of speeds, although **the average kinetic energy of the sample depends only on temperature**

Maxwell-Boltzmann Molecular Speed Distribution for Noble Gases
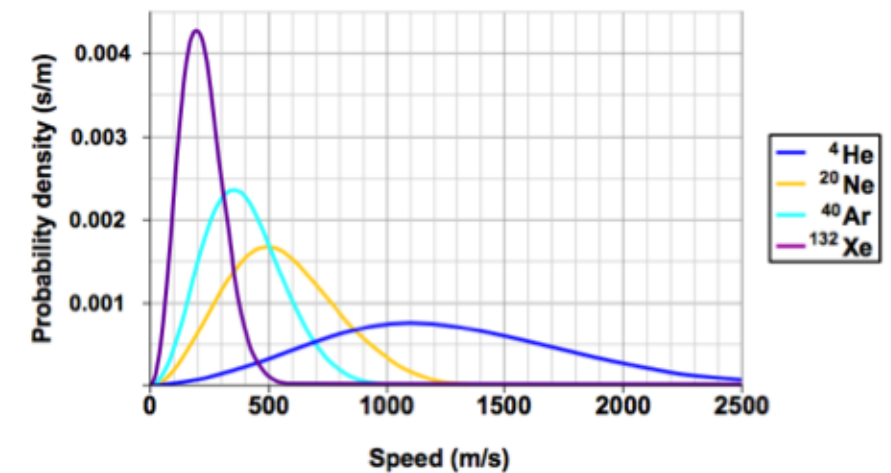
Summary:

- **Ideal gases are pretty easy if you use the ideal gas law properly.**
  - Students who get these problems wrong usually do one of two things:
    - Their R value is wrong
    - They do not convert from Celsius to Kelvin

- **Stoichiometry is important for gases, so learn it**
  - Remember to differentiate between questions asking for the quantity of product and the quantity of total species in the final system

- **The average kinetic energy of a system is only dependent on the temperature**

- Coming up:
  - The equations of KMT
  - Mixtures of gases: partial pressure and concentrations
  - Deviating from ideal gases: understanding that gas molecules have volume and there are very small but non-negligible interactions between them
  - Exam!

Kinetic Molecular Theory: Relationships

- Kinetic Molecules Theory gives us three key relationships that you should know as equations and by definition (in words)

1. Kinetic Energy vs. Temperature

$$E_k = \frac{3}{2}RT$$ ← **Energy per mole (J/mol)**

- **Kinetic energy is dependent solely on the temperature of a gaseous system (direct relationship)**
- R = 8.314 J / mol K

$$E_k = \frac{3}{2}kT$$ ← **Energy per molecule (J)**

- $k = \dfrac{R}{N_a}$

2. Mass vs. Velocity

3. Temperature vs. Velocity

Remember that in physics we describe kinetic energy with the equation:

$$\frac{1}{2}mv^2$$

KMT: Mass vs. Velocity; Velocity vs. Temperature

1. Kinetic Energy vs. Temperature

Based on the equation:

$$V_{rms} = \sqrt{\frac{3RT}{M}}$$

R = 8.314 J/mol K

Molecular Weight in Kg

We can determine that velocity is proportional to the **square root of temperature** and the **inverse square root** of mass.

2. **Mass vs. Velocity (V_rms)**
   ◦ Velocity is proportional to the inverse square root of mass.
   ◦ When temperature is constant, lighter particles move faster

$$\frac{v_1}{v_2} = \sqrt{\frac{M_2}{M_1}}$$

Pay very close attention to how the 1 and 2 match up in these relationships

3. **Velocity (V_rms) vs. Temperature**
   ◦ Velocity is proportional to the square of temperature
   ◦ When dealing with the same species gas, particles move faster at higher temperatures

$$\frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}}$$

Practice Problem

What is the ratio of the effusion rates of SO_2 to Cl_2?
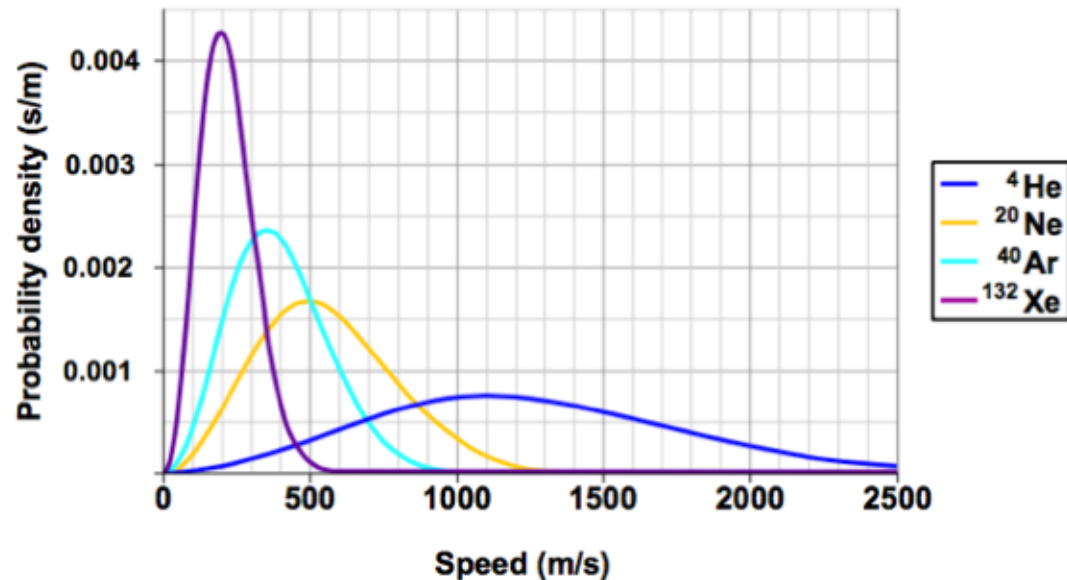
$$\frac{v_{SO_2}}{v_{Cl_2}} = \sqrt{\frac{M_{Cl_2}}{M_{SO_2}}}$$

The position of these is important because it is asking you for the ratio of $SO_2 : Cl_2$

If the velocity of Cl_2 is 500m/s, what is the temperature?

$$V_{SO_2} = \sqrt{\frac{3RT}{M}}$$

KMT: Mass vs. Velocity; Velocity vs. Temperature

Maxwell-Boltzmann Molecular Speed Distribution for Noble Gases

Legend:
- ^{4}He
- ^{20}Ne
- ^{40}Ar
- ^{132}Xe

Y-axis: Probability density (s/m) — 0.001, 0.002, 0.003, 0.004
X-axis: Speed (m/s) — 0, 500, 1000, 1500, 2000, 2500

1. **Mass vs. Velocity (V_{rms})**
   - We can see that the heavier gases move slower and the lighter gases move faster
   - The faster the gas, the wider the distribution

2. **Velocity (V_{rms}) vs. Temperature**
   - If you were working with the same gas, a similar graph could be created by modifying temperature instead.
   - In this case, higher temperatures of the same gas result in faster speeds.

- Some key features of this graph include:
  - Each curve looks like a unimodal distribution with a "tail" that approaches the limit infinite velocity (0 probability density)
  - Molecules are traveling at a variety of speeds but there is a clear average
  - The actual V_{rms} is slightly to the right of the peak

Maxwell-Boltzmann Distribution

Maxwell-Boltzmann Molecular Speed Distribution for Noble Gases
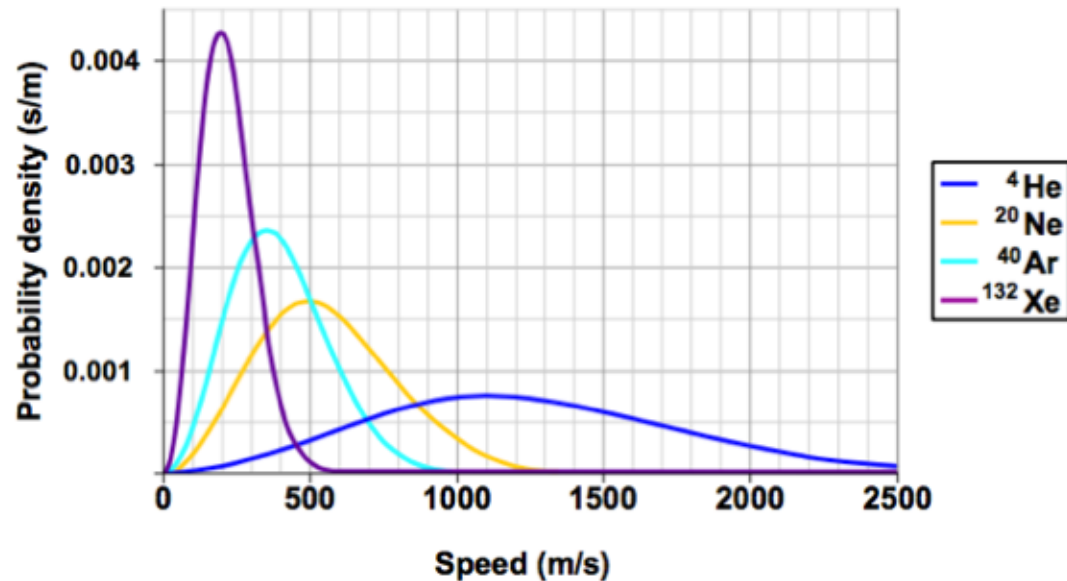
- Given a Maxwell distribution, you should know:
  - Find the approximate V_{rms}
  - Be able to label **which gas is which** (or which temperature is which)
  - Understand the **relationships** between mass, velocity, and temperature
  - Understand how these relationships impact the **shape** of the curve